ADDENDA AND CORRIGENDA

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- Addendum to [1].

  - In the proof of Proposition 5.2 of [1], the correct reference is to Theorem 5.6 of [18] (instead of Theorem 5.5 of [18]).

- Corrigenda to [2].

  - In Proposition 2.6, the polynomials F_i are in $n+2$ variables (instead of in $n+1$ variables).
  - In Proposition 4.3.3, the condition "irreducible" on $h(f)$ should be replaced by the condition "absolutely irreducible". Indeed, this is used in the proof when saying that there are at most $d(d-1)$ lines L in the direction of v (for the directional derivative not to vanish identically). At every use of this Proposition 4.3.3 in the paper, this extra condition of absolutely irreducibility for $h(f)$ is already satisfied. The same correction should be made to Proposition 7.1 of [Marcelo Paredes, Román Sasyk, *Uniform bounds for the number of rational points on varieties over global fields*, ANT, 2022].
  - In Proposition 4.3.4, the upper bound $cd^{14}B$ for $N_{\mathrm{aff}}(f,B)$ should be $cd^{18}B$. Indeed, in the proof of Proposition 4.3.4, δ can be at most d times the expression from (4.3.1) (this factor d is forgotten in the reasoning). The exponent 18 has been picked up correctly in Proposition 7.3 of [Marcelo Paredes, Román Sasyk, *Uniform bounds for the number of rational points on varieties over global fields*, ANT, 2022]. This correction is now obsolete in view of the improvements made in [6].
  - In Lemma 4.3.7, the degree should be $(n+1)d(d^2-1)$ instead of $(n+1)(d^2-1)$. The corresponding correction should be made in Lemma 7.5 of [Marcelo Paredes, Román Sasyk, *Uniform bounds for the number of rational points on varieties over global fields*, ANT, 2022].
  - Remark 4.3.8 should be changed according to the corrected versions of Proposition 4.3.4 and Lemma 4.3.7. This correction is now obsolete in view of the improvements made in [6].

- Addendum to [3] and [8].

- In Remark 2.1.16 of [5], an additional reduction argument is explained that was used but not made explict in [3] and [8].

- Corrigendum to [4].
  - In [4], there are two alternative definitions given for cones with multiplicities and it is claimed that they are equivalent, namely the equality of $CM_0^\Lambda(X)$ and $SC_0^\Lambda(X)$ in Section 5.4 of [4]. In fact, they are not equal in general, and, only $CM_0^\Lambda(X)$ should be used throughout [4] (and not $SC_0^\Lambda(X)$), as noted in Section 6.1 of [19]. Even with this corrected definition, the final part of Theorem 3.6.2 of [4] remains unproved as far as we know, and Proposition 2.4.2 of [4] is wrong as stated, as noted in Section 6.1 of [19]. We thank Arthur Forey for pointing this out to us.

- Corrigenda to [7].
  - In Lemma 5.1.3, Proposition 5.1.4 and Theorems B and 5.2.2 of [7], for L of characteristic zero, the dependence of the bounds on the degree ν of L over $\mathbb{Q}_p$ is forgotten. For Lemma 5.1.3 this is explained in [11]. Put $\nu = 1$ if L is a of positive characteristic, and otherwise let ν be the degree of L over $\mathbb{Q}_p$. In Theorem 5.2.2 of [7], the constant C, the integer ℓ and the family W should be allowed to depend additionally on ν. In Theorem B of [7], the constant C and the set W^ε should be allowed to depend additionally on ν. In Proposition 5.1.4, the constants C and C' should be allowed to depend additionally on ν.
  - On line 8 in Section 1.2 of [7] there is a typo: t should be send to ϖ_L instead of to φ_L.
  - The obviously intended inclusion $W_{y,s} \subset X_y$ should be added explicitly in Theorem 5.2.2 of [7] and the same for the inclusion $V_{y,\sigma} \subset Z_y$ in Theorem 4.2.3 of [5].

- Corrigendum to [9].
  - In Proposition 4.1 of [9] one should add the extra condition on the coefficients $c_{a,b,i,j}$ that they are non-torsion whenever $a \neq 0$. This is explained in [15].

- Corrigenda to [10].
  - A smoothness condition is missing in Proposition 4.3.1 of [10]; this smoothness condition is made explicit and explained in Remark 6.6 of [1].
  - In Definition 2.1.1 of [10], the norm is missing in the numerator of the difference quotient.

- Addenda to [12].
  - In [13, Remark A.1.8], a correction to Theorem 4.5.15 of [12] is made. Namely, it is noted that the order symbol $<$ should be part of the language $\mathcal{L}_\mathcal{A}$ in Definition 4.5.14 to make the statement of Theorem 4.5.15 correct.
  - In [13, Remark A.1.9], it is explained that one of the two proofs given for the quantifier elimination statement of Theorem 6.3.7 of [12] is incomplete, and this proof is completed in Remark A.1.9. One proof uses compactness, and, the other proof is more classical and follows the line of quantifier elimination

results going back to [17] and [21]. We thank Silvain Rideau for pointing this out to us.

- In Definition 4.1.2(ii) of [12], it should be added that $A_{m,n}$ is closed under permutation of the ξ-variables and also under permutation of the ρ-variables. Similarly, in Definition 4.3.1(ii) of [12], it should be added that A_m is closed under permutation of the ξ-variables. In Definition 3.1.1 of [12] it is optional to add it or not (the results of the paper are correct without adding the stability under permutation of the variables). Note that in [13, Definition 3.1.2(ii)], the stability under permutation is already explicitely mentioned, and hence, no correction is needed.

- Addendum to [14].

- In the whole paper [14] the base field k is assumed to be of characteristic zero. This should have been made more explicit, but is implicitly clear from the context and used results.

- Addendum to [16].

- In Definition 3.3 of [16], the power series F has to converge on an open neighborhood of the **closure** of the image of φ. This word 'closure' should be added, in line with previous preparation results as in [20] [23] [22].

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